

Higgs Mechanism with Type-II Nambu-Goldstone Bosons at Finite Chemical Potential

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When the spontaneous symmetry breaking occurs for systems without Lorentz covariance, there arises possible mismatch, $N_{\text{NG}} < N_{\text{BG}}$, between numbers of Nambu-Goldstone (NG) bosons (N_{NG}) and the numbers of broken generators (N_{BG}). In such a situation, so-called type-II NG bosons emerge. We study how the gauge bosons acquire masses through the Higgs mechanism under this mismatch by employing gauge theories with complex scalar field at finite chemical potential and by enforcing “charge” neutrality. To separate the physical spectra from unphysical ones, the R_ξ gauge is adopted. Not only massless NG bosons but also massive scalar bosons generated by the chemical potential are absorbed into spatial components of the gauge bosons. Although the chemical potential induces a non-trivial mixings among the scalar bosons and temporal components of the gauge bosons, it does not affect the structure of the physical spectra, so that the total number of physical modes is not modified even for $N_{\text{NG}} < N_{\text{BG}}$.

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I. INTRODUCTION

The spontaneous symmetry breaking (SSB) and the Higgs mechanism are the two key concepts in both elementary particle physics and condensed matter physics. One of the most important aspects of SSB is the appearance of massless Nambu-Goldstone (NG) bosons [1, 2]: In particular, for systems with Lorentz covariance, the number of NG bosons, N_{NG} , is equal to the number of broken generators, N_{BG} , of the symmetry group under consideration [3]. If the symmetry is local, these NG bosons are absorbed into the gauge bosons and disappear from the physical spectra [4]. However, for the system without Lorentz covariance, there arise situations with $N_{\text{NG}} \neq N_{\text{BG}}$: A well-known example is the Heisenberg ferromagnet where there is only one NG magnon while the number of broken generator associated with $O(3) \rightarrow O(2)$ is two, i.e. $N_{\text{NG}} < N_{\text{BG}}$ (see e.g. [5]). This is in contrast to the Heisenberg antiferromagnet which shows the same symmetry breaking pattern but has two magnons, i.e. $N_{\text{NG}} = N_{\text{BG}}$ (See TABLE I).

It was realized by Nielsen and Chadha [6] that such a mismatch between N_{NG} and N_{BG} as the ferromagnet is related to the dispersion relation of the NG bosons. By introducing type-I and type-II NG bosons according to whether the dispersion relation is proportional to odd and even powers of momentum in the long wavelengths, they have shown an inequality, $N_{\text{I}} + 2 \times N_{\text{II}} \geq N_{\text{BG}}$ where N_{I} (N_{II}) is the total numbers of type-I (type-II) NG bosons. The magnon in the antiferromagnet (ferromagnet) is type-I (type-II), in this classification. The kaon condensation in the color-flavor-locked (CFL) phase of high density quantum chromodynamics (QCD) shows another example of this mismatch. It is a relativistic system with Lorentz covariance explicitly broken by chemical potential and has both type-I and type-II NG bosons

[7] (See also, [8]). An important role of the commutation relations among broken generators for the emergence of the type-II NG bosons was also realized in this context as reviewed in Ref. [9].

A natural question to ask in the presence of local gauge symmetry is the fate of the gauge bosons and Higgs mechanism with type-II NG bosons. For systems with $N_{\text{NG}} = N_{\text{I}} = N_{\text{BG}}$, the number of massive gauge bosons (except for the spin degrees of freedom) due to Higgs mechanism is equal to the number of broken generators. On the other hand, for the systems with $N_{\text{NG}} < N_{\text{BG}}$, it is not entirely obvious how the Higgs mechanism works and what would remain in the physical spectra at low energies. In this paper, we study the Higgs mechanism with type-II NG bosons in relativistic systems that Lorentz covariance is explicitly broken by chemical potential. In our analysis, we employ gauge theories with complex scalar field at finite chemical potential such as gauged SU(2) model, Glashow-Weinberg-Salam type gauged U(2) model, and gauged SU(3) model, which are known to have both type-I and type-II NG bosons if gauge couplings are absent. To ensure the non-Abelian charge neutrality of the system, we introduce non-Abelian external sources according to [10]. Then, we derive explicitly the mass spectra of the scalar bosons and gauge bosons in the tree level. To separate physical spectra from unphysical ones clearly, we adopt the R_ξ gauge with the gauge parameter taken as infinity at the end.¹

¹ In ref.[11], the similar problem was treated without imposing the non-Abelian charge neutrality. In such an approach, the temporal component of the gauge field acquires non-vanishing expectation value in contrast to ours. This leads to the dispersion relations of the physical modes and the behavior of the system near the weak gauge-coupling limit different from ours

system	SSB-pattern	N_{BG}	N_{NG}	NG boson	dispersion relation
2-flavor QCD	$\text{SU}(2)_{\text{L}} \times \text{SU}(2)_{\text{R}} \rightarrow \text{SU}(2)_{\text{V}}$	3	3	pion	$E(p) \propto p$
Heisenberg antiferromagnet	$\text{O}(3) \rightarrow \text{O}(2)$	2	2	magnon	$E(p) \propto p$
Heisenberg ferromagnet	$\text{O}(3) \rightarrow \text{O}(2)$	2	1	magnon	$E(p) \propto p^2$

TABLE I: Examples of SSB. N_{NG} and N_{BG} denote the total number of NG bosons and broken generators, respectively.

This paper is organized as follows. In Sec. II, we briefly review NG boson spectra at finite chemical potential by taking the U(2) model of complex scalar fields. In Sec. III, we delineate how the Higgs mechanism including the type-II NG bosons works through this model by gauging the SU(2) symmetry. The Glashow-Weinberg-Salam type gauged U(2) model is also studied. In Sec. IV, we discuss U(3) model with its SU(3) part gauged as a toy model for the two-flavor color superconductivity (2SC) in dense QCD. In all these models, we introduce the chemical potential for U(1) (hyper) charge. Section V is devoted to summary and concluding remarks. In Appendix, we make a detailed comparison of the results of Higgs mechanism at finite chemical potential with and without the background charge density by taking the gauged SU(2) model as an example.

II. TYPE-II NG BOSONS IN U(2) MODEL

Let us first review the NG boson spectra at finite chemical potential through the U(2) model of complex scalar fields. It was originally introduced as a model for kaon condensation in the color-flavor-locked (CFL) phase of high density QCD [7]. The Lagrangian of the U(2) model is given by

$$\mathcal{L} = |(\partial_0 - i\mu)\phi|^2 - |\partial_i\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4, \quad (1)$$

with m^2 and λ being positive, and $\phi = (\phi_1, \phi_2)^t$ denoting 2-component complex scalar field. Equation (1) possesses U(2) ($\cong \text{SU}(2) \times \text{U}(1)$) symmetry, $\phi' = [\exp(-i\theta_\alpha \tau^\alpha)] \phi$, ($\alpha = 0, 1, 2, 3$), where τ^α s are the U(2) generators. Under the SSB pattern $\text{U}(2) \rightarrow \text{U}(1)_Q$ ($Q = \frac{1}{2}(1 + \tau_3)$) obtained by the ground state expectation values, $\langle \phi_1 \rangle = 0$ and $\langle \phi_2 \rangle = v/\sqrt{2}$ with $v = \sqrt{(\mu^2 + m^2)/\lambda}$, we may parametrize the scalar field as

$$\begin{aligned} \phi &= \frac{1}{\sqrt{2}} \left[v + \psi + i \sum_{a=1}^3 \chi_a \tau^a \right] \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \\ &= \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_2 + i\chi_1 \\ v + \psi - i\chi_3 \end{pmatrix}. \end{aligned} \quad (2)$$

(see Appendix for details).

Quadratic part of the Lagrangian for the fluctuation fields reads

$$\begin{aligned} \mathcal{L}_0 &= \frac{1}{2}(\partial_\mu \chi_a)^2 + \frac{1}{2}[(\partial_\mu \psi)^2 - 2(\mu^2 + m^2)\psi^2] \\ &\quad - \mu(\chi_3 \overleftrightarrow{\partial}_0 \psi + \chi_2 \overleftrightarrow{\partial}_0 \chi_1), \end{aligned} \quad (3)$$

with a notation, $A \overleftrightarrow{\partial}_0 B \equiv A \partial_0 B - (\partial_0 A) B$. Then the equations of motion for ψ and χ_a are given by

$$\begin{bmatrix} \partial_0^2 - \partial_i^2 & -2\mu\partial_0 \\ 2\mu\partial_0 & \partial_0^2 - \partial_i^2 \end{bmatrix} \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = 0, \quad (4)$$

$$\begin{bmatrix} \partial_0^2 - \partial_i^2 + 2(\mu^2 + m^2) & -2\mu\partial_0 \\ 2\mu\partial_0 & \partial_0^2 - \partial_i^2 \end{bmatrix} \begin{bmatrix} \psi \\ \chi_3 \end{bmatrix} = 0. \quad (5)$$

Solving these equations in momentum space leads to the dispersion relations:

$$E_{\chi_1-\chi_2} = \sqrt{p^2 + \mu^2} \pm \mu = \begin{cases} \frac{p^2}{2\mu} + O(p^4), \\ 2\mu + O(p^2). \end{cases} \quad (6)$$

$$\begin{aligned} E_{\chi_3-\psi} &= \left[p^2 + \frac{m_\psi^2}{2} \pm \frac{m_\psi^2}{2} \sqrt{1 + \frac{16\mu^2 p^2}{m_\psi^2}} \right]^{1/2} \\ &= \begin{cases} \sqrt{\frac{\mu^2 + m^2}{3\mu^2 + m^2}} p + O(p^2), \\ \sqrt{6\mu^2 + 2m^2} + O(p^2), \end{cases} \end{aligned} \quad (7)$$

where $m_\psi \equiv \sqrt{6\mu^2 + 2m^2}$. These dispersion relations are shown in Fig. 1. From the mixing between ψ and χ_3 induced by the chemical potential μ , one massive mode ψ' and one massless mode χ'_3 arise. The latter is the type-I NG boson whose energy is proportional to p . On the other hand, from the mixing between χ_1 and χ_2 induced by μ , one massive mode χ'_1 and one massless mode χ'_2 arise. The latter is the type-II NG boson whose energy is proportional to p^2 in the low-momentum limit. Although we have $N_{\text{NG}} = 2$ which is smaller than $N_{\text{BG}} = 3$, the Nielsen-Chadha relation is satisfied as an equality:

$$N_{\text{I}} + 2 \times N_{\text{II}} = 1 + 2 \times 1 = N_{\text{BG}}. \quad (8)$$

III. GAUGED SU(2) MODEL AT FINITE μ

In this section, by gauging the SU(2) part of the U(2) model introduced in the previous section, we discuss the

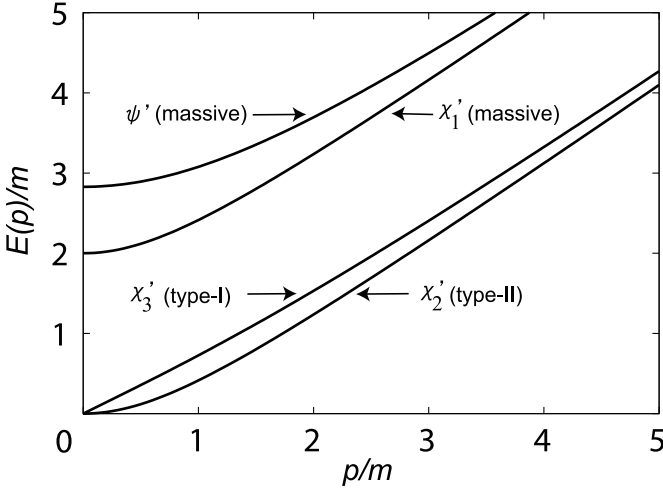


FIG. 1: Dispersion relation for the fluctuation fields in the case of $\mu/m = 1$. Due to the mixing induced by the chemical potential, there arise only two NG bosons instead of three; one is the type-I (χ_3') with $E \propto p$ and the other is type-II (χ_2') with $E \propto p^2$ at low momentum.

Higgs mechanism at finite chemical potential with a type-II NG boson. Fate of the gauge bosons with only two NG bosons is of our central interest here as we mentioned in the Introduction. The Lagrangian of the gauged SU(2) model with finite chemical potential is given by

$$\mathcal{L} = -\frac{1}{4}(F_a^{\mu\nu})^2 + |(D^0 - i\mu)\phi|^2 - |D^i\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4 + gj_\mu^a A_a^\mu, \quad (9)$$

where $F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu + g\epsilon_{abc}A_b^\mu A_c^\nu$ and $D^\mu = \partial^\mu - i\frac{g}{2}\tau_a^a A_a^\mu$ with g and τ^a ($a = 1, 2, 3$) being the gauge coupling and SU(2) generators, respectively. $j_\nu^a = j_0^a \delta_{\nu 0}$ is a background non-Abelian charge density to ensure the charge neutrality [10].

We take the same parametrization as Eq. (2) for the scalar fields and adopt the gauge condition (the R_ξ gauge),

$$F_a = \frac{1}{\sqrt{\alpha}}(\partial_\mu A_a^\mu + M\alpha\chi_a) \quad (a = 1, 2, 3), \quad (10)$$

with $M = \frac{g}{2}v = \frac{g}{2}\sqrt{(\mu^2 + m^2)/\lambda}$ and α being the gauge parameter. The chemical potential μ is embedded in F_a implicitly through M . An advantage of taking the R_ξ gauge is that one can clearly separate the physical and unphysical degrees of freedom; masses of unphysical particles go to infinity and decouple from physical particles in the limit $\alpha \rightarrow \infty$. As we see shortly, this is particularly useful to analyze the situation with new mixing terms induced by the chemical potential.

With the above gauge condition, the quadratic part of

the Lagrangian with the ghost fields (c_a and \bar{c}_a) reads

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu)^2 + \frac{1}{2}M^2(A_a^\mu)^2 - \frac{1}{2\alpha}(\partial_\mu A_a^\mu)^2 \\ & + \frac{1}{2}[(\partial_\mu \psi)^2 - 2(\mu^2 + m^2)\psi^2] \\ & + i\bar{c}_a(\partial^2 + \alpha M^2)c_a + \frac{1}{2}[(\partial_\mu \chi_a)^2 - \alpha M^2\chi_a^2] \\ & - \mu(\chi_3 \overleftrightarrow{\partial}_0 \psi + \chi_2 \overleftrightarrow{\partial}_0 \chi_1) \\ & - 2\mu M(-\chi_2 A_1^{\nu=0} + \chi_1 A_2^{\nu=0} + \psi A_3^{\nu=0}). \end{aligned} \quad (11)$$

The first three lines in Eq. (11) are the standard Lagrangian in the R_ξ gauge except for the implicit μ dependence in M . The fourth line is a mixing induced by the chemical potential which leads to the type-II NG boson as discussed in Sec. II. The fifth line is a mixing of gauge fields with massless and massive scalar bosons induced by the chemical potential. Note here that the linear term of the gauge field, $-\mu M v A_3^{\nu=0}$, arising from $|(D_0 - i\mu)\phi|^2$ is cancelled by the background charge contribution $gj_\nu^a A_a^\nu$ with $gj_\nu^a = \mu M v \delta_{\nu 0}^a$.

From Eq. (11), one finds that the spatial components of the gauge field $A_a^{\nu=1,2,3}$ absorb not only massless NG bosons but also massive scalar boson generated by the chemical potential and acquire the mass M . On the other hand, the temporal component of the gauge fields $A_a^{\nu=0}$ and the scalar fields (χ_a and ψ) mix with each other through the chemical potential. Therefore, it is important to check how the mixing affects the physical and unphysical spectra of the system. For this purpose, we take $p = 0$ and examine the equations of motion obtained from Eq. (11),

$$\begin{bmatrix} E^2 - \alpha M^2 & -2i\mu E & 0 & -2\mu\sqrt{\alpha}M \\ 2i\mu E & E^2 - \alpha M^2 & 2\mu\sqrt{\alpha}M & 0 \\ 0 & -2\mu\sqrt{\alpha}M & E^2 - \alpha M^2 & 0 \\ 2\mu\sqrt{\alpha}M & 0 & 0 & E^2 - \alpha M^2 \end{bmatrix}$$

$$\times \begin{bmatrix} \chi_1 \\ \chi_2 \\ \frac{1}{\sqrt{\alpha}}A_1^{\nu=0} \\ \frac{1}{\sqrt{\alpha}}A_2^{\nu=0} \end{bmatrix} \equiv \mathcal{M}_1 \vec{X} = 0, \quad (12)$$

$$\begin{bmatrix} E^2 - 2(\mu^2 + m^2) & -2i\mu E & -2\mu\sqrt{\alpha}M \\ 2i\mu E & E^2 - \alpha M^2 & 0 \\ 2\mu\sqrt{\alpha}M & 0 & E^2 - \alpha M^2 \end{bmatrix} \times \begin{bmatrix} \psi \\ \chi_3 \\ \frac{1}{\sqrt{\alpha}}A_3^{\nu=0} \end{bmatrix} \equiv \mathcal{M}_2 \vec{Y} = 0. \quad (13)$$

Equation (12) implies that $\chi_{1,2}$ and $A_a^{\nu=0}$ have a large diagonal (mass)² of $O(\alpha)$ for large α . Also there are off-diagonal terms of $O(\sqrt{\alpha})$, i.e. $\pm 2i\mu E \simeq \pm 2i\mu M\sqrt{\alpha}$ (for the χ_1 - χ_2 mixing) and $\pm 2\mu\sqrt{\alpha}M$ (for the χ_1 - $\frac{1}{\sqrt{\alpha}}A_2^{\nu=0}$ and χ_2 - $\frac{1}{\sqrt{\alpha}}A_1^{\nu=0}$ mixings). By approximating E by $\sqrt{\alpha}M$

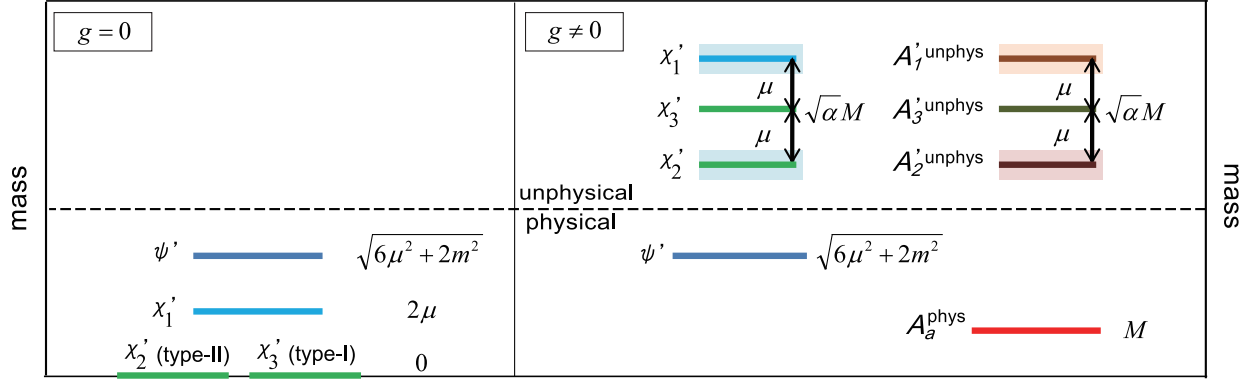


FIG. 2: The left side shows the mass spectrum at $g = 0$ where gauge field and the scalar fields do not interact. The physical particles are a type-I NG boson (χ'_3), a type-II NG boson (χ'_2), a former NG boson (χ'_1), and a Higgs scalar (ψ). In addition, there are six (transverse $\times 3$) massless gauge bosons. The right side shows the mass spectrum at $g \neq 0$ where the SU(2) gauge symmetry is spontaneously broken. The physical modes are the nine ((transverse+longitudinal) $\times 3$) massive gauge bosons $A_a^{\nu=1,2,3}$ with a mass M and the Higgs scalar ψ' . Others are unphysical modes with the mass of about $\sqrt{\alpha}M$.

in the off-diagonal terms (which is justified for large α), one can solve $\det \mathcal{M}_1 = 0$ and obtain

$$E_{\chi_{1,2}-A_{1,2}^0} = \sqrt{\alpha}M \left(1 \pm \frac{2\mu}{\sqrt{\alpha}M} e^{\pm i\pi/3} \right)^{1/2}. \quad (14)$$

This result shows that both $\chi_{1,2}$ and $A_{1,2}^{\nu=0}$ decouple from physical particles due to their large masses of $O(\sqrt{\alpha})$ with a small and complex mass splittings of $O(\mu)$. For Eq. (13), $\det \mathcal{M}_2 = 0$ can be solved exactly as

$$E_{\chi_3-A_3^0} = \sqrt{\alpha}M, \quad (15)$$

$$E_{\psi'} = \sqrt{6\mu^2 + 2m^2}. \quad (16)$$

This shows that $A_3^{\nu=0}$ and χ_3 decouple from physical particles due to their large masses of $O(\sqrt{\alpha})$, while ψ remains as a physical particle with a mass not modified at all by the mixing due to μ .

Taken together, there arise six unphysical modes: not only the type-I and type-II NG bosons ($\chi'_{2,3}$) but also a massive mode (χ'_1) become unphysical together with $A_{1,2,3}^{\nu=0}$ due to Higgs mechanism. On the other hand, the physical modes are the spatial components of the gauge field $A_a^{\nu=1,2,3}$ (two transverse and one longitudinal) with the mass M , and the scalar mode ψ' with its mass remaining invariant under the mixing induced by the chemical potential. The schematic illustration of the mass spectra is shown in FIG. 2. Numbers of physical particles with and without the gauge coupling g are listed in TABLE II; the total physical degrees of freedom (=10) are conserved regardless of the Higgs mechanism at finite μ .

Let us briefly discuss the Glashow-Weinberg-Salam type gauged U(2) (\cong SU(2) \times U(1)_Y) model with the SU(2) gauge fields, A_a^μ , and the U(1)_Y gauge field, B^μ . Qualitative aspects of the Higgs mechanism at finite μ in this case is the same as that of the gauged SU(2) model.

chemical potential	$\mu \neq 0$	
gauge coupling	$g = 0$	$g \neq 0$
gauge bosons	2×3	3×3
NG bosons	2 (type-I & II)	0
massive bosons	2	1

TABLE II: Comparison of the physical degrees of freedom at finite μ with and without the gauge coupling g .

The mixing term induced by μ in the Glashow-Weinberg-Salam model reads

$$\begin{aligned} \mathcal{L}_0^{\text{mix}} = & -\mu(\chi_3 \overleftrightarrow{\partial}_0 \psi + \chi_2 \overleftrightarrow{\partial}_0 \chi_1) \\ & -2\mu M_W (-\chi_2 W_1^{\mu=0} + \chi_1 W_2^{\mu=0}) \\ & -2\mu M_Z \psi Z^{\mu=0}, \end{aligned} \quad (17)$$

where $M_W = M = \frac{g}{2}v$, $M_Z = \frac{(g^2+g'^2)^{1/2}}{2}v$, g' denoting the U(1)_Y gauge coupling, $W_{1,2}^\mu = A_{1,2}^\mu$, and $Z^\mu = (g^2+g'^2)^{-1/2}(gA_3^\mu - g'B^\mu)$. We impose both Abelian and non-Abelian charge neutralities by introducing the U_Y(1) background charge density ($g'B^\mu j_\mu$) and the SU(2) background charge density ($gA_a^\mu j_\mu^a$). They have a role to cancel out the tadpole term of gauge field, $-\mu M_Z v Z^{\nu=0}$. Comparing these with Eq. (11), the only modification is the effect of g' , which leads the χ'_3 mass to be $\sqrt{\alpha}M_Z$. Physical mass spectra are essentially the same as FIG. 2 with an addition of massless photon field.

IV. GAUGED SU(3) MODEL AT FINITE μ

In this section, we discuss the Higgs mechanism in the gauged SU(3) model at finite μ whose Lagrangian is given by the same form as Eq. (9) with ϕ replaced

by a 3-component complex scalar field and τ^a replaced by the SU(3) generators. If we interpret ϕ as colored diquarks, the SU(3) gauge fields as gluons and the background source $gj_\mu^a A_a^\mu$ as a contribution from unpaired quarks, this Lagrangian may be considered as a toy model for the two-flavor color superconductivity (2SC) in dense QCD [12]. This explains the physical meaning of the non-Abelian background charge density we have introduced.

Let us parametrize ϕ as

$$\begin{aligned}\phi &= \frac{1}{\sqrt{2}}[v + \psi + i \sum_{a=4}^8 \chi_a \tau^a] \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} \chi_5 + i\chi_4 \\ \chi_7 + i\chi_6 \\ v + \psi - i\frac{2}{\sqrt{3}}\chi_8 \end{bmatrix}. \quad (18)\end{aligned}$$

By adopting the R_ξ gauge and choosing the background charge as $gj_a^\nu = \frac{2}{\sqrt{3}}\mu M v \delta_{a8}^{\nu=0}$ to maintain neutrality, the mixing term induced by μ in the quadratic part of the Lagrangian becomes

$$\begin{aligned}\mathcal{L}_0^{\text{mix}} &= -\mu \left(\frac{2}{\sqrt{3}}\chi_8 \overleftrightarrow{\partial}_0 \psi + \chi_5 \overleftrightarrow{\partial}_0 \chi_4 + \chi_7 \overleftrightarrow{\partial}_0 \chi_6 \right) \\ &\quad - 2\mu M (-\chi_5 A_4^{\nu=0} + \chi_4 A_5^{\nu=0} - \chi_7 A_6^{\nu=0} + \chi_6 A_7^{\nu=0} \\ &\quad + \frac{2}{\sqrt{3}}\psi A_8^{\nu=0}). \quad (19)\end{aligned}$$

If the gauge coupling g is zero, there arise three massive scalar bosons ψ' and $\chi'_{4,6}$, two type-II NG bosons $\chi'_{5,7}$, and one type-I NG boson χ'_8 due to the mixing among scalar fields. The mixing terms between the scalar fields and the temporal component of the gauge fields induced by the chemical potential lead to the similar equation of motion as the SU(2) case at $p = 0$,

$$\mathcal{M}_1 \begin{bmatrix} \chi_4 \\ \chi_5 \\ \frac{1}{\sqrt{\alpha}} A_4^{\nu=0} \\ \frac{1}{\sqrt{\alpha}} A_5^{\nu=0} \end{bmatrix} = \mathcal{M}_1 \begin{bmatrix} \chi_6 \\ \chi_7 \\ \frac{1}{\sqrt{\alpha}} A_6^{\nu=0} \\ \frac{1}{\sqrt{\alpha}} A_7^{\nu=0} \end{bmatrix} = 0, \quad (20)$$

$$\mathcal{M}_2 \begin{bmatrix} \psi \\ \frac{2}{\sqrt{3}}\chi_8 \\ \frac{2}{\sqrt{3\alpha}} A_8^{\nu=0} \end{bmatrix} = 0. \quad (21)$$

By applying the same argument as given in Eq. (12), $\chi_{4,5,6,7,8}$ and $A_{4,5,6,7,8}^{\nu=0}$ are found to decouple from physical particles due to their large masses of $O(\sqrt{\alpha})$. On the other hand, the spatial component of the gauge field $A_a^{\nu=1,2,3}$ with a mass $M = \frac{g}{2}\sqrt{(\mu^2 + m^2)}/\lambda$ and the scalar mode ψ' with a mass $m_\psi = \sqrt{6\mu^2 + 2m^2}$ remain as physical particles. Total physical degrees of freedom in this case (=16) are conserved regardless of the Higgs mechanism at finite μ .

V. SUMMARY AND CONCLUSIONS

In this paper, we studied how the Higgs mechanism with type-II NG bosons works at finite chemical potential μ by imposing the Abelian and non-Abelian charge neutrality. We adopt a relativistic U(2) model of two component scalar field which exhibits both type-I and type-II NG bosons due to the mixing term induced by the chemical potential. By gauging the SU(2) part of this model and adopting the R_ξ gauge, we examined the physical and unphysical modes of the system. The result is schematically shown in FIG. 2: The type-I NG boson, the type-II NG boson, and one of the massive scalar boson which was a type-I NG boson at $\mu = 0$ are absorbed in the gauge bosons to create a gauge boson mass $M = \frac{g}{2}\sqrt{(\mu^2 + m^2)}/\lambda$. The mass of the Higgs scalar was found to receive no effect despite that it mixes with the gauge boson and the type-I NG boson due to chemical potential. As a result, total physical degrees of freedom are conserved regardless of the Higgs mechanism at finite μ .

We applied the above analysis to the the gauged U(2) model (Glashow-Weinberg-Salam type model) and the gauged SU(3) model (a toy model for the 2SC in dense QCD) at finite μ . Essential features were found to be the same as those in the gauged SU(2) model. Generalization to the U(N) model with the SSB pattern $U(N) \rightarrow U(N-1)$, which has one type-I NG boson and $N-1$ type-II NG bosons, is rather straightforward. In this paper, we analyzed Higgs mechanism with type-II NG bosons in relativistic systems that Lorentz covariance is explicitly broken by chemical potential. It will be an interesting future problem to extend the present analysis for intrinsically nonrelativistic systems with type-II NG bosons such as Heisenberg ferromagnet.

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Appendix

In this Appendix, we compare our results in Sec. III and those of ref. [11] by taking gauged SU(2) model as an example. As mentioned in Sec. I, the difference between two approaches originates from the treatment of non-Abelian charge neutrality.

Before starting the comparison, we first review the case of the gauged U(1) model at finite chemical potential

following ref. [10]. The Lagrangian density is given by

$$\mathcal{L} = -\frac{1}{4}(F^{\mu\nu})^2 + |(D^0 - i\mu)\phi|^2 - |D^i\phi|^2 + m^2|\phi|^2 - \lambda|\phi|^4, \quad (22)$$

where $D^\mu = \partial^\mu - i\frac{g'}{2}A^\mu$ with g' being the U(1) gauge coupling. Let us determine the ground state of the system without the background charge density. (The gauge fixing condition such as the R_ξ gauge does not affect the conclusion.) The equations of motion for scalar field ϕ and gauge boson A^μ are

$$-(\tilde{D}_\nu \tilde{D}^\nu - m^2)\phi = 2\lambda(\phi^\dagger \phi)\phi, \quad (23)$$

$$\partial_\mu F^{\mu\nu} = -i\frac{g'}{2}\phi^\dagger \overleftrightarrow{\partial}^\nu \phi - g' \left(\frac{g'}{2}A^\nu + \mu\delta^{\nu 0} \right) \phi^\dagger \phi, \quad (24)$$

where $\tilde{D}^\nu = D^\nu - i\mu\delta^{\nu 0}$. By solving the above equations in the mean field approximation, and denoting the ground state expectation value of the scalar field as $\langle\phi\rangle = \phi_0/\sqrt{2} = \text{const.} \neq 0$, we have $\langle A^{\nu=i}\rangle = 0$ and

$$\left(\frac{g'}{2}\langle A^{\nu=0}\rangle + \mu \right)^2 + m^2 = \lambda\phi_0^2, \quad (25)$$

$$\left(\frac{g'}{2}\langle A^{\nu=0}\rangle + \mu \right) \phi_0^2 = 0. \quad (26)$$

Thus we obtain

$$\phi_0^2 = \frac{m^2}{\lambda}, \quad \langle A^\nu \rangle = -2\frac{\mu}{g'}\delta^{\nu 0}. \quad (27)$$

Expanding the fields around the minimums, $\phi = (\phi_0 + \psi + i\chi)/\sqrt{2}$, $A^\nu = \mathcal{A}^\nu + \langle A^\nu \rangle$, the quadratic part of the Lagrangian becomes

$$\mathcal{L}_0 = -\frac{1}{4}(\mathcal{F}^{\mu\nu})^2 + \frac{1}{2}M_0'^2(\mathcal{A}^\mu - M_0'^{-1}\partial^\mu\chi)^2 + \frac{1}{2}[(\partial_\mu\psi)^2 - 2m^2\psi^2], \quad (28)$$

with $\mathcal{F}_{\mu\nu} = \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu$ and $M_0' = \frac{g'}{2}\phi_0$. We find that the chemical potential μ disappears from \mathcal{L}_0 . This unphysical situation is due to the absence of the background charge density $g'j_\mu A^\mu$ [10].

Introducing the background charge density to Eq. (22), the mean-field equations (25) and (26) are modified as

$$\langle A^\nu \rangle = 0, \quad (29)$$

$$\mu^2 + m^2 = \lambda v^2, \quad (30)$$

$$\frac{1}{2}\mu v^2 + j_{\nu=0} = 0, \quad (31)$$

with v defined by $\langle\phi\rangle \equiv v/\sqrt{2}$. We thus find that

$$v^2 = \frac{\mu^2 + m^2}{\lambda}, \quad \langle A^\nu \rangle = 0, \quad j_\nu = -\frac{1}{2}\mu v^2 \delta_{\nu 0}. \quad (32)$$

The quadratic part of the Lagrangian for the fluctuation fields ψ , χ and $A(=\mathcal{A})$ reads

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}(F^{\mu\nu})^2 + \frac{1}{2}M'^2(A^\mu - M'^{-1}\partial^\mu\chi)^2 \\ & + \frac{1}{2}[(\partial_\mu\psi)^2 - 2(\mu^2 + m^2)\psi^2] \\ & + \mu(\chi_3 \overleftrightarrow{\partial}_0 \psi) + 2\mu M'\psi A^{\nu=0} \\ & + \mu M'v A^{\nu=0} + g'j_\mu A^\mu, \end{aligned} \quad (33)$$

with $M' = \frac{g'}{2}v$. The total charge density of the system is the sum of condensation charge and the background charge which cancel with each other:

$$\begin{aligned} \rho_{\text{tot}} &= \left\langle \frac{\partial \mathcal{L}_0}{\partial A^{\nu=0}} \right\rangle \\ &= g' \left(\frac{1}{2}\mu v^2 + j_{\nu=0} \right) = 0. \end{aligned} \quad (34)$$

The masses of the fluctuation fields (15) and (16) are obtained from \mathcal{L}_0 by employing the R_ξ gauge. We note that these masses approach smoothly to those in (7) by taking the limit, $g' \rightarrow 0$.

Now let us generalize the above discussion to the gauged SU(2) model. We first study the model without the background charge density following [11]. In this case, the Lagrangian is given by (9) without the term $gj_\mu^a A_a^\mu$. Then the equations of motions for scalar field ϕ and gauge bosons A_a^μ become

$$-(\tilde{D}_\nu \tilde{D}^\nu - m^2)\phi = 2\lambda(\phi^\dagger \phi)\phi, \quad (35)$$

$$(\mathcal{D}_\mu F^{\mu\nu})^a = -ig\phi^\dagger \frac{\tau^a}{2} \overleftrightarrow{\partial}^\nu \phi - g\phi^\dagger \left(\frac{g}{2}A_a^\nu + \mu\delta_{\nu 0}\tau^a \right) \phi. \quad (36)$$

Solving the above equations in the mean field approximation with $\langle\phi\rangle = (0, \phi_0/\sqrt{2})$, we have

$$\langle A_\pm^{\nu=0} \rangle = 0, \quad (37)$$

$$\left(\frac{g}{2}\langle A_3^{\nu=0} \rangle - \mu \right)^2 + m^2 = \lambda\phi_0^2, \quad (38)$$

$$\left(\frac{g}{2}\langle A_3^{\nu=0} \rangle - \mu \right) \phi_0^2 = 0, \quad (39)$$

with $A_\mp^\mu = (A_1^\mu \pm iA_2^\mu)/\sqrt{2}$. Then the ground states are characterized by the condensations,

$$\phi_0^2 = \frac{m^2}{\lambda}, \quad \langle A_a^\nu \rangle = 2\frac{\mu}{g}\delta_{a3}^{\nu 0}. \quad (40)$$

The quadratic part of Lagrangian for the fluctuation fields becomes

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}(\mathcal{F}_a^{\mu\nu})^2 + \frac{1}{2}M_0^2(A_a^\mu - M_0^{-1}\partial^\mu\chi_a)^2 \\ & - 2\mu(\mathcal{F}_{0i}^2\mathcal{A}_1^i - \mathcal{F}_{0i}^1\mathcal{A}_2^i) + 2\mu^2((\mathcal{A}_1^i)^2 + (\mathcal{A}_2^i)^2) \\ & + \frac{1}{2}[(\partial_\mu\psi)^2 - 2m^2\psi^2] \\ & - 2\mu(\chi_2 \overleftrightarrow{\partial}_0 \chi_1) + 2\mu^2(\chi_1^2 + \chi_2^2) \\ & - 2\mu M(-\chi_2\mathcal{A}_1^{\nu=0} + \chi_1\mathcal{A}_2^{\nu=0}), \end{aligned} \quad (41)$$

with $\mathcal{F}_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a$ and $M_0 = \frac{g}{2}\phi_0$.

Even without the background charge density, the SU(2) charge neutrality is still ensured by the gauge bosons having nonzero expectation value, and we have $\rho_{\text{tot}}^a = \langle \frac{\partial \mathcal{L}_0}{\partial A_\nu^a} \rangle = 0$. Furthermore, dispersion relations for $A_{\pm,3}^{\mu=i}$ and ψ become

$$E_{A_{\mp}^{\mu=i}}^2 = \left(\sqrt{p^2 + (g\phi_0/2)^2} \pm 2\mu \right)^2, \quad (42)$$

$$E_{A_3^{\mu=i}}^2 = p^2 + (g\phi_0/2)^2, \quad (43)$$

$$E_\psi^2 = p^2 + 2m^2. \quad (44)$$

The magnitude of the condensate of the gauge field in eq. (40) grows as g becomes small, so that the phase characterized by eq. (40) is distinct from the ground state of the non-gauged U(2) model in Sec. II. Accordingly, the dispersion relation for the Higgs boson in (44) does not approach to eq. (7) in the limit $g \rightarrow 0$, and the gauge bosons $A_{\mp}^{\mu=i}$ are not massless in the limit $g = 0$.

We now turn to the ground state of the system with the addition of SU(2) background charge density, $gj_\mu^a A_\mu^a$, as discussed in Sec. in Sec. III. By solving the Lagrangian given by (9) in the mean field approximation with $\langle \phi \rangle = (0, v/\sqrt{2})$, we obtain

$$\langle A_a^\nu \rangle = 0, \quad (45)$$

$$\mu^2 + m^2 = \lambda v^2, \quad (46)$$

$$-\frac{1}{2}\mu v^2 + j_{\nu=0}^3 = 0. \quad (47)$$

Thus we obtain

$$v^2 = \frac{\mu^2 + m^2}{\lambda}, \quad \langle A^\nu \rangle = 0, \quad j_\nu^a = \frac{1}{2}\mu v^2 \delta_{\nu 0}^{a3}. \quad (48)$$

The quadratic part of the Lagrangian for the fluctuation fields reads

$$\begin{aligned} \mathcal{L}_0 = & -\frac{1}{4}(F_{\mu\nu}^a)^2 + \frac{1}{2}M^2(A_a^\mu - M^{-1}\partial^\mu \chi_a)^2 \\ & + \frac{1}{2}[(\partial_\mu \psi)^2 - 2(\mu^2 + m^2)\psi^2] - \mu(\chi_3 \overleftrightarrow{\partial}_0 \psi + \chi_2 \overleftrightarrow{\partial}_0 \chi_1) \\ & - 2\mu M(-\chi_2 A_1^{\nu=0} + \chi_1 A_2^{\nu=0} + \psi A_3^{\nu=0}) \\ & - \mu M v A_3^{\nu=0} + g j_\nu^a A_a^\nu. \end{aligned} \quad (49)$$

In this case, the SU(2) charge neutrality is ensured by the cancellation between the condensation charge and the background charge:

$$\rho_{\text{tot}}^{a=3} = \langle \frac{\partial \mathcal{L}_0}{\partial A_3^{\nu=0}} \rangle = g \left[-\frac{1}{2}\mu v^2 + j_{\nu=0}^{a=3} \right] = 0, \quad (50)$$

$$\rho_{\text{tot}}^{a=1,2} = 0. \quad (51)$$

Adopting the R_ξ gauge, and solving the equations of motions at $p = 0$, we obtain Eqs. (12) and (13). These equations in the limit $g \rightarrow 0$ reproduce the masses of the scalar fields (6) and (7). Therefore the phase characterized by Eq. (48) is smoothly connected to the ground state of the non-gauged U(2) model in Sec. II.

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- [1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), *ibid.* **124**, 246 (1961).
 - [2] J. Goldstone, Nuovo Cim. **19**, 154 (1961).
 - [3] J. Goldstone, A. Salam and S. Weinberg, Phys. Rev. **127**, 965 (1962).
 - [4] Y. Nambu, Phys. Rev. **117**, 648 (1960); P. W. Anderson, Phys. Rev. **130**, 439 (1963); F. Englert and R. Brout, Phys. Rev. Lett. **13**, 321 (1964); P. W. Higgs, Phys. Lett. **12**, 132 (1964); Phys. Rev. Lett. **13**, 508 (1964); G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. **13**, 585 (1964).
 - [5] C. Kittel, *Quantum Theory of Solids*, 2nd. edition (Wiley, 1987); H. Leutwyler, Phys. Rev. D **49**, 3033 (1994) [arXiv:hep-ph/9311264].
 - [6] H. B. Nielsen and S. Chadha, Nucl. Phys. B **105**, 445 (1976).
 - [7] V. A. Miransky and I. A. Shovkovy, Phys. Rev. Lett. **88**, 111601 (2002) [arXiv:hep-ph/0108178]. T. Schäfer,
 - D. T. Son, M. A. Stephanov, D. Toublan and J. J. M. Verbaarschot, Phys. Lett. B **522**, 67 (2001) [arXiv:hep-ph/0108210].
 - [8] D. Blaschke, D. Ebert, K. G. Klimenko, M. K. Volkov and V. L. Yudichev, Phys. Rev. D **70**, 014006 (2004) [arXiv:hep-ph/0403151].
 - [9] T. Brauner, Symmetry **2**, 609 (2010) [arXiv:1001.5212 [hep-th]].
 - [10] J. I. Kapusta, Phys. Rev. D **24**, 426 (1981);
 - [11] V. P. Gusynin, V. A. Miransky and I. A. Shovkovy, Phys. Lett. B **581**, 82 (2004) [arXiv:hep-ph/0311025].
 - [12] K. Iida and G. Baym, Phys. Rev. D **66**, 014015 (2002) [arXiv:hep-ph/0204124]; I. Giannakis and H.-c. Ren, Phys. Rev. D **65**, 054017 (2002) [arXiv:hep-ph/0108256]; M. Eto, M. Nitta and N. Yamamoto, Phys. Rev. Lett. **104**, 161601 (2010) [arXiv:0912.1352 [hep-ph]].